# Solid Mechanics - 202041

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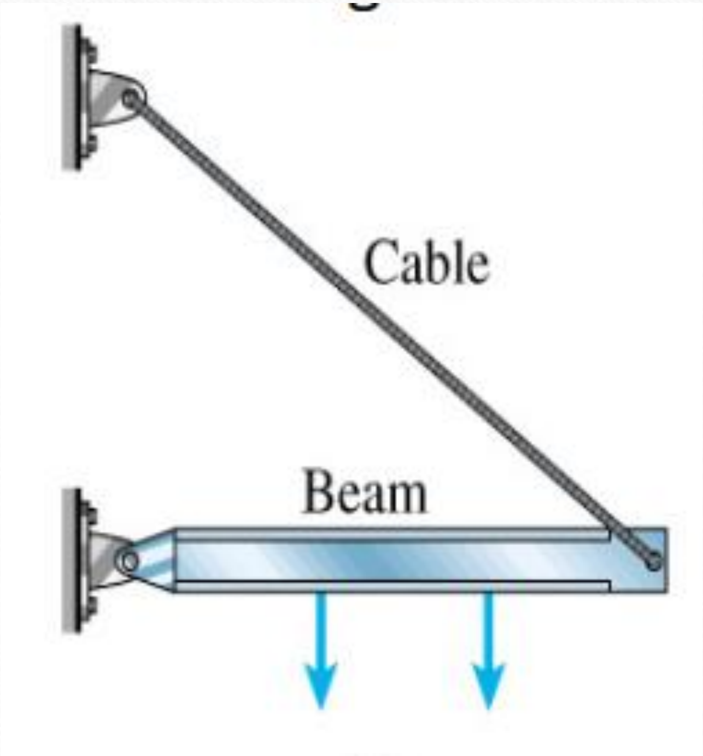
# Unit VI Application based combined loading & stresses

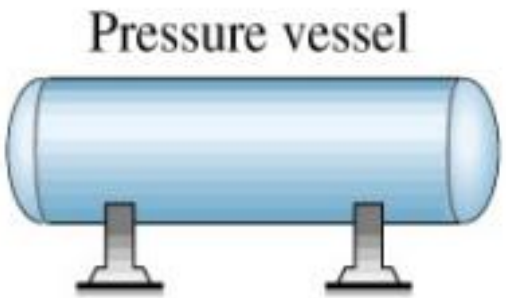
- Introduction to the Combined Loading and various stresses with application, Free Body Diagram and condition of Equilibrium for determining internal reaction forces, couples for 2-D system, Combined stresses at any cross-section or at any particular point for Industrial and Real life example for the following cases: Combined problem of Normal type of Stresses (Tensile, Compressive and Bending stress), Combined problem of Shear type of stresses (Direct and Torsional Shear stresses), Combined problem of Normal and Shear type of Stresses

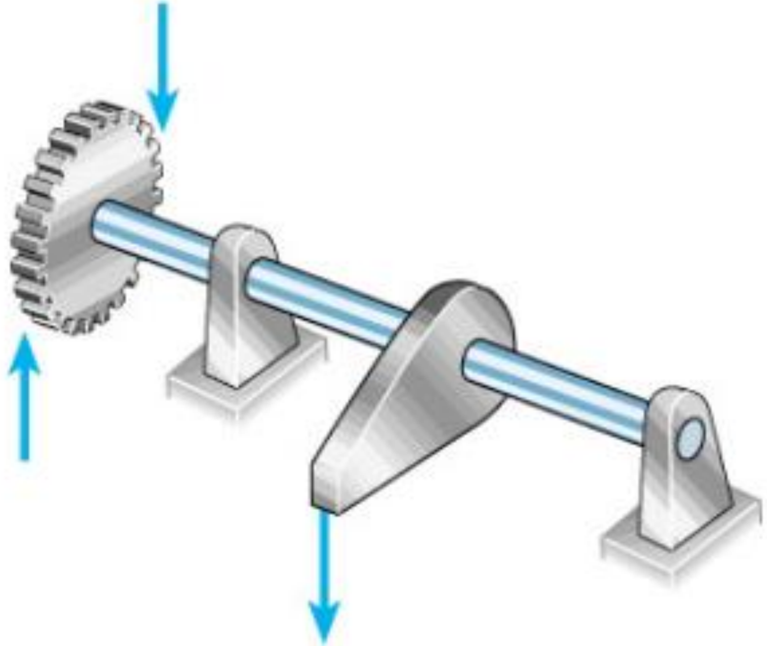
Direct stress alone is produced in a body when it is subjected to an axial tensile or compressive load. And bending stress is produced in the body, when it is subjected to a bending moment. But if a body is subjected to axial loads and also bending moments, then both the stresses (*i.e.*, direct and bending stresses) will be produced in the body.

# Introduction to the Combined Loading and various stresses with application

# Introduction to the Combined Loading and various stresses with application



## Method of Analysis:

1.Select the point on the structure where the stresses and the strains are to be determined.

2.For each load on the structure, determine the stress resultant at the cross section containing the selected point..

3.Calculate the normal and shear stresses at the selected point due to each of the stress resultant.

4.Combine the individual stresses to obtain the resultant stresses at the selected point.

5.Determine the principal stresses and maximum shear stresses at the selected point.

6.Determine the strains at the point with the aid of Hooke’s law for plane stress.

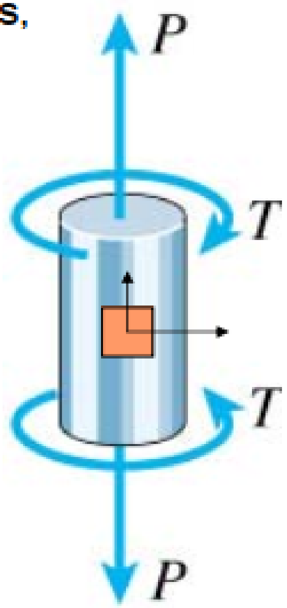
7.Select additional points and repeat the process.

$$\sigma = \frac{P}{A} \qquad \tau = \frac{T\rho}{I_\rho} \qquad \sigma = -\frac{My}{I}$$

$$\tau = \frac{VQ}{Ib} \qquad \sigma = \frac{pr}{t}$$

**Example** The rotor shaft of an helicopter drives the rotor blades that provide the lifting force to support the helicopter in the air. As a consequence, the shaft is subjected to a combination of torsion and axial loading.

For a ***50mm*** diameter shaft transmitting a torque ***T = 2.4kN.m*** and a tensile force ***P = 125kN***, determine the maximum tensile stress, maximum compressive stress, and maximum shear stress in the shaft.




**Solution**

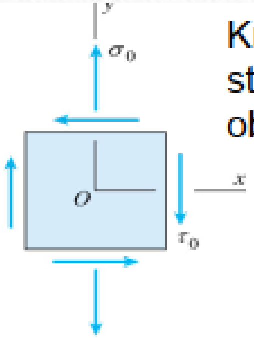
The stresses in the rotor shaft are produced by the combined action of the axial force ***P*** and the torque ***T***. Therefore the stresses at any point on the surface of the shaft consist of a tensile stress $\sigma_o$ and a shear stress $\tau_o$.

The tensile stress

$$\sigma = \frac{P}{A} = \frac{125kN}{\pi/4\,(0.05m)^2} = 63.66MPa$$

The shear stress to is obtained from the torsion formula

$$\tau_{Torsion} = \frac{Tr}{I_P} = \frac{(2.4kN.m)\left(\frac{0.05}{2}\right)}{\frac{\pi(0.05)^4}{32}} = 97.78MPa$$




Knowing the stresses σo and τo, we can now obtain the principal stresses and maximum shear stresses . The principal stresses are obtained from

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \left(\frac{0 + 63.66}{2}\right) \pm \sqrt{\left(\frac{0 - 63.66}{2}\right)^2 + (-97.78)^2}$$

$$\sigma_1 = 135MPa$$

$$\sigma_2 = -71MPa$$

The maximum in-plane shear stresses are obtained using the formula

Because the principal stresses $\sigma_1$ and $\sigma_2$ have opposite signs, the maximum in-plane shear stresses are larger than the maximum out-of-plane shear stresses. Therefore, the maximum shear stress in the shaft is 103MPa.

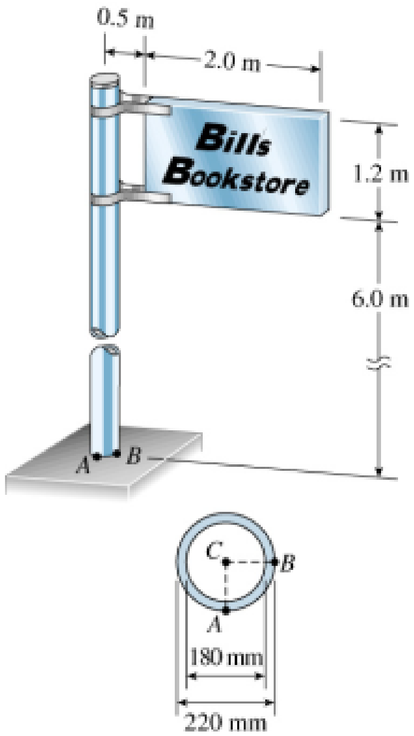
Will it fail if $\sigma_{yield}$=480MPa?

$$MSST \Rightarrow SF = \frac{480MPa/2}{103MPa} = 2.33$$

$$\sigma_{VM} = \sqrt{(135)^2 - (135)(-71) + (-71)^2} = 181.2MPa$$

$$DET \Rightarrow SF = \frac{480MPa}{181.2MPa} = 2.65$$

Example




A sign of dimensions ***2.0mx1.2m*** is supported by a hollow circular pole having outer diameter ***220mm*** and inner diameter ***180mm*** (see figure). The sign offset ***0.5m*** from the centerline of the pole and its lower edge is ***6.0m*** above the ground.

Determine the principal stresses and maximum shear stresses at points ***A*** and ***B*** at the base of the pole due to wind pressure of ***2.0kPa*** against the sign.

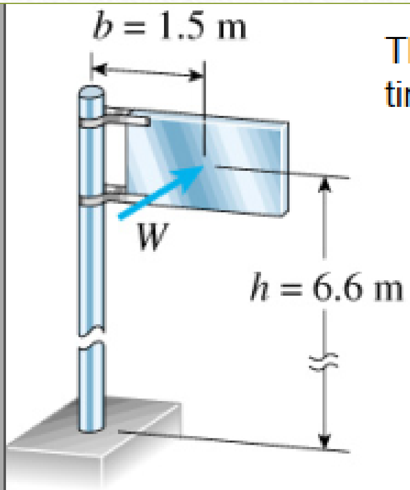
**Solution**

*Stress Resultant:* The wind pressure against the sign produces a resultant force ***W*** that acts at the midpoint of the sign and it is equal to the pressure ***p*** times the area ***A*** over which it acts:

$$W = pA = (2.0kPa)(2.0m \times 1.2m) = 4.8kN$$

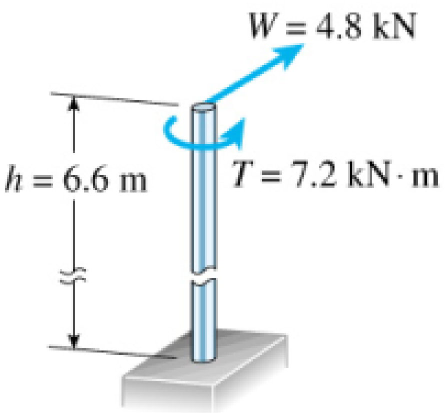
The line of action of this force is at height ***h = 6.6m*** above the ground and at distance ***b = 1.5m*** from the centerline of the pole.

The wind force acting on the sign is statically equivalent to a lateral force ***W*** and a torque ***T*** acting on the pole.




The torque is equal to the force ***W*** times the distance ***b***:

$$T = Wb = (4.8kN)(1.5m)$$
$$T = 7.2kN - m$$




The stress resultant at the base of the pole consists of a bending moment M, a torque T and a shear force V. Their magnitudes are:

M = Wh = (4.8kN)(6.6m) = 31.68kN.m

T = 7.2kN.m

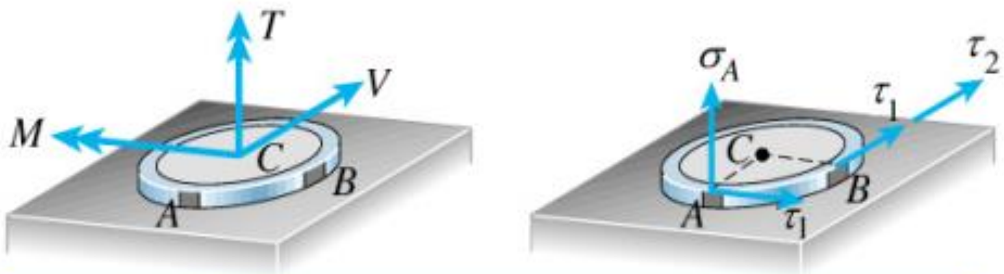
V = W = 4.8kN

Examination of these stress resultants shows that maximum bending stresses occur at point ***A*** and maximum shear stresses at point ***B***.

Therefore, ***A*** and ***B*** are critical points where the stresses should be determined.

***Stresses at points A and B***

The bending moment ***M*** produces a tensile stress $\sigma_a$ at point ***A***, but no stress at point ***B*** (which is located on the neutral axis)




$$\sigma_a = \frac{M\left(\frac{d_2}{2}\right)}{\left[\frac{\pi\left(d_2^4 - d_1^4\right)}{64}\right]} = \frac{(31.68kN)(0.11m)}{\left[\frac{\pi\left(0.22^4 - 0.18^4\right)}{64}\right]} = 54.91MPa$$

The torque ***T*** produces shear stresses $\tau_1$ at points ***A*** and ***B***.

$$\tau_{Torsion} = \frac{T\left(\frac{d_2}{2}\right)}{\left[\frac{\pi\left(d_2^4 - d_1^4\right)}{32}\right]} = \frac{(7.2kN.m)(0.11m)}{\left[\frac{\pi\left(0.22^4 - 0.18^4\right)}{32}\right]} = 6.24MPa$$

Finally, we need to calculate the direct shear stresses at points ***A*** and ***B*** due to the shear force ***V***.

The shear stress at point ***A*** is zero, and the shear stress at point ***B*** ($\tau_2$) is obtained from the shear formula for a circular tube

$$\tau_{2,Max} = \frac{2V}{A} = \frac{2(4800)}{0.01257m^2} = 0.7637MPa$$

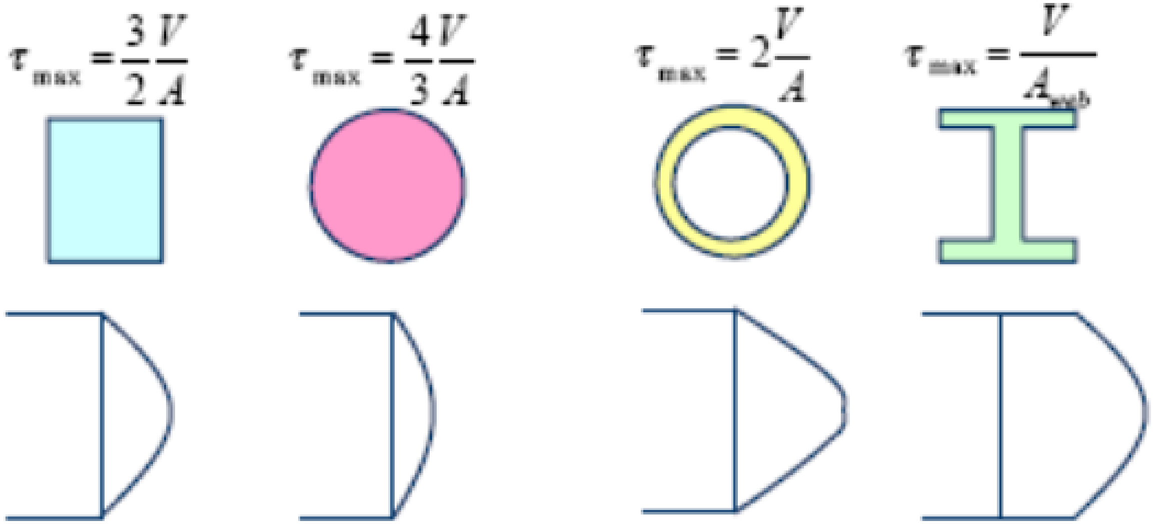
$$\tau_2 = \frac{VQ}{Ib}$$

$$I = \left[\frac{\pi\left(d_2^4 - d_1^4\right)}{64}\right]$$

$$Q = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

$$b = 2(r_2 - r_1)$$

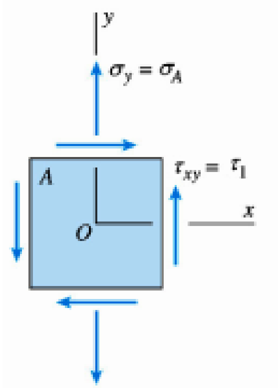
The stresses acting on the cross section at points ***A*** and ***B*** have now been calculated.



## Stress Elements

For both elements the y-axis is parallel to the longitudinal axis of the pole and the x-axis is horizontal.

### Point A :




$\sigma_x = 0$

$\sigma_y = \sigma_a = 54.91MPa$

$\tau_{xy} = \tau_1 = 6.24MPa$

Principal stresses at Point ***A***

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Substituting $\sigma_{1,2} = 27.5MPa \pm 28.2MPa$

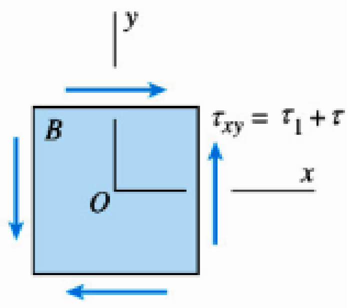
***$\sigma_1 = 55.7MPa$ and $\sigma_2 = -0.7MPa$***

The maximum in-plane shear stresses can be obtained from the equation

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = 28.2MPa$$

Because the principal stresses have opposite signs, the ***maximum in-plane shear stresses are larger than the maximum out-of-plane shear stresses***.

Then, ***$\tau_{max} = 28.2MPa$.***




**Point B :**

$\sigma_x = \sigma_y = 0$

$\tau_{xy} = \tau_1 + \tau_2$

$\tau_{xy}$ = 6.24MPa + 0.76MPa = 7.0MPa

Principal stresses at point ***B*** are

***$\sigma_1$ = 7.0MPa***      ***$\sigma_2$ = - 7.0 MPa***

And the maximum in-plane shear stress is

***$\tau_{max}$ = 7.0MPa***

The maximum out-of-plane shear stresses are half of this value.

***Note***

If the largest stresses anywhere in the pole are needed, then we must also determine the stresses at the critical point diametrically opposite point ***A***, because at that point the compressive stress due to bending has its largest value.

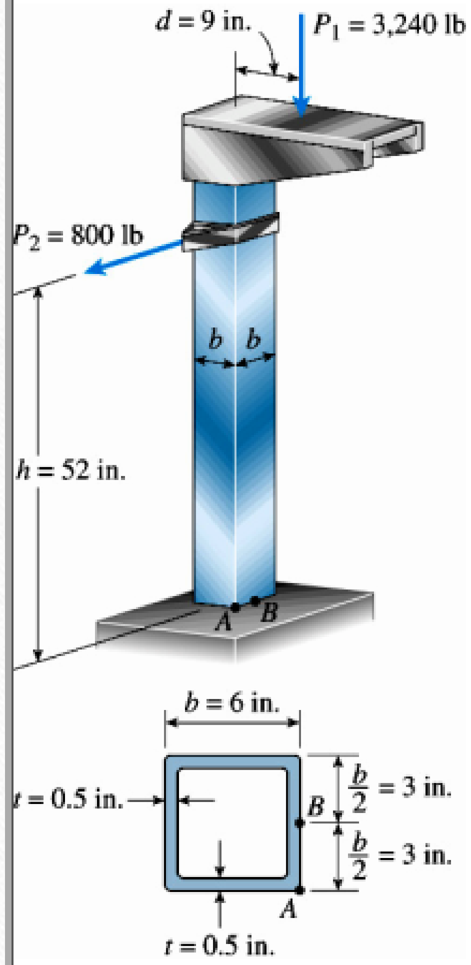
The principal stresses at that point are

***$\sigma_1$ = 0.7MPa***      ***and***      ***$\sigma_2$ = - 55.7MPa***

The maximum shear stress is *28.2MPa.*

*(In this analysis only the effects of wind pressure are considered. Other loads, such as weight of the structure, also produce stresses at the base of the pole).*

## Example




A tubular post of square cross section supports a horizontal platform (see figure).

The tube has outer dimension ***b* = *6in***.

And wall thickness ***t* = *0.5in***.

The platform has dimensions ***6.75in x 24.0in*** and supports an uniformly distributed load of ***20psi*** acting over its upper surface.

The resultant of this distributed load is a vertical force

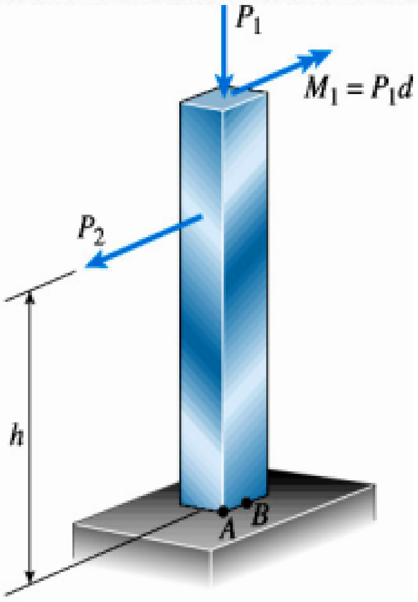
$P_1$ = (20psi)(6.75in x 24.0in) = 3240lb

This force acts at the midpoint of the platform, which is at distance

***d* = *9in***. from the longitudinal axis of the post.

A second load ***$P_2$* = *800lb*** acts horizontally on the post at height ***h* = *52in*** above the base.

Determine the principal stresses and maximum shear stresses at points A and B at the base of the post due to the loads $P_1$ and $P_2$.




### Solution

*Stress Resultants*

The force $\boldsymbol{P_1}$ acting on the platform is statically equivalent to a force $\boldsymbol{P_1}$ and a moment $\boldsymbol{M_1 = P_1 d}$ acting on the centroid of the cross section of the post.

The load $\boldsymbol{P_2}$ is also shown.

The stress resultant at the base of the post due to the loads $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ and the moment $\boldsymbol{M_1}$ are as follows:

(A) An axial compressive force $\boldsymbol{P_1}$ = 3240lb

(B) A bending moment $\boldsymbol{M_1}$ produced by the force $\boldsymbol{P_1}$:

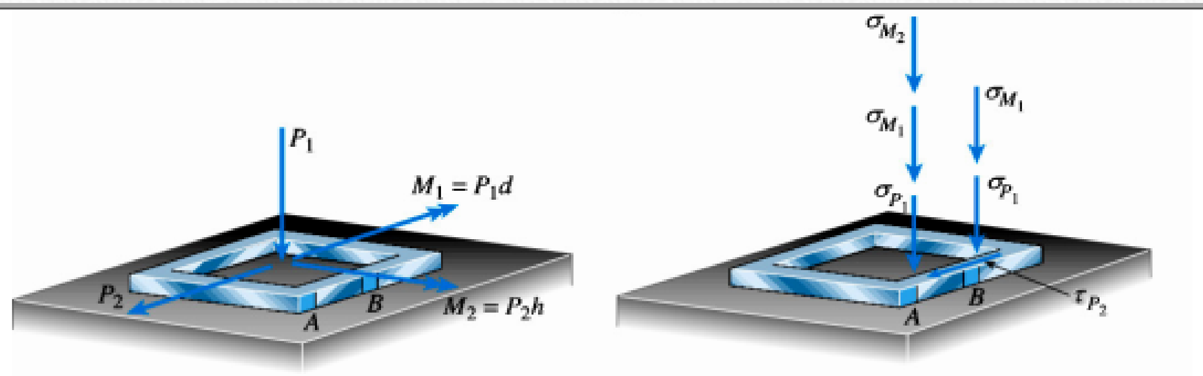
$$M_1 = P_1 d = (3240\text{lb})(9\text{in}) = 29160\text{lb-in}$$

(C) A shear force $\boldsymbol{P_2}$ = 800lb

(D) A bending moment $\boldsymbol{M_2}$ produced by the force $\boldsymbol{P_2}$:

$$M_2 = P_2 h = (800\text{lb})(52\text{in}) = 41600\text{lb.in}$$

***Examinations of these stress resultants shows that both $M_1$ and $M_2$ produce maximum compressive stresses at point A and the shear force produces maximum shear stresses at point B. Therefore, A and B are the critical points where the stresses should be determined.***




***Stresses at points A and B***

(A) The axial force $\boldsymbol{P_1}$ produces uniform compressive stresses throughout the post. These stresses are $\boldsymbol{\sigma_{P1} = P_1 / A}$ where $\boldsymbol{A}$ is the cross section area of the post

$\boldsymbol{A = b^2 - (b - 2t)^2 = 4t(b-t)}$ = 4 (0.5in)(6in – 0.5in) = 11.0in$^2$

$\boldsymbol{\sigma_{P1} = P_1 / A}$ = 3240lb / 11.00in$^2$ = 295psi

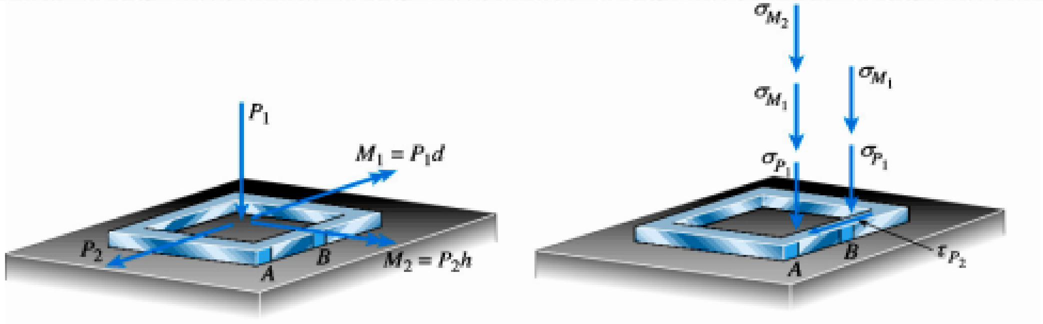
(B) The bending moment $\boldsymbol{M_1}$ produces compressive stresses $\boldsymbol{\sigma_{M1}}$ at points $\boldsymbol{A}$ and $\boldsymbol{B}$. These stresses are obtained from the flexure formula

$\boldsymbol{\sigma_{M1} = M_1 (b / 2) / I}$

where $I$ is the moment of inertia of the cross section. The moment of inertia is

$\boldsymbol{I = [b^4 - (b - 2t)^4] / 12}$ = [(6in)$^4$ – (5in)$^4$] / 12 = 55.92in$^4$

Thus, $\boldsymbol{\sigma_{M1} = M_1 b / 2I}$ = (29160lb.in)(6in) / (2)(55.92in$^4$) = 1564psi




(C) The shear force $P_2$ produces a shear stress at point ***B*** but not at point ***A***. We know that an approximate value of the shear stress can be obtained by dividing the shear force by the web area.

$\tau_{P2} = P_2 / A_{web} = P_2 /(2t(b - 2t))$ =800lb / (2)(0.5in)(6in–1in)= 160psi

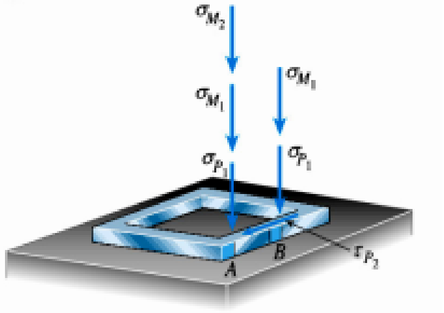
The stress $\tau_{p2}$ acts at point ***B*** in the direction shown in the above figure.

We can calculate the shear stress $\tau_{P2}$ from the more accurate formula. The result of this calculation is $\tau_{P2}$ = 163psi, which shows that the shear stress obtained from the approximate formula is satisfactory.

D) The bending moment $M_2$ produces a compressive stress at point ***A*** but no stress at point ***B***. The stress at ***A*** is

$\sigma_{M2} = M_2\, b / 2I$ = (41600lb.in)(6in) / (2)(55.92in$^4$) = 2232psi.

This stress is also shown in the above figure.




*Stress Elements*

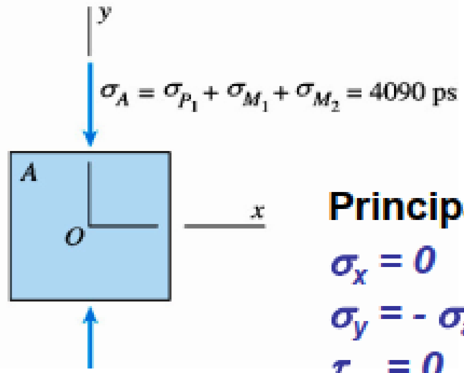
Each element is oriented so that the ***y***-axis is vertical (i.e. parallel to the longitudinal axis of the post) and the ***x***-axis is horizontal axis

**Point A :** The only stress in point ***A*** is a compressive stress $\sigma_a$ in the ***y*** direction

$\sigma_A = \sigma_{P1} + \sigma_{M1} + \sigma_{M2}$

$\sigma_A$ = 295psi + 1564psi + 2232psi = 4090psi (compression)

Thus, this element is in uniaxial stress.




**Principal Stresses and Maximum Shear Stress**

$\sigma_x = 0$

$\sigma_y = -\sigma_a = -4090psi$

$\tau_{xy} = 0$

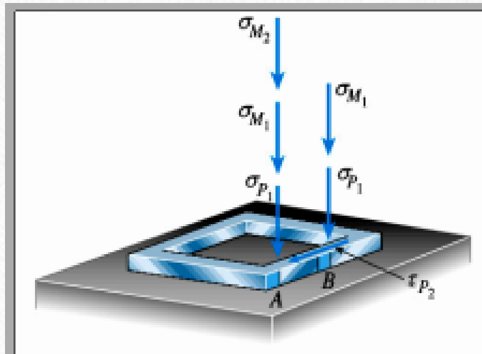
Since the element is in uniaxial stress,

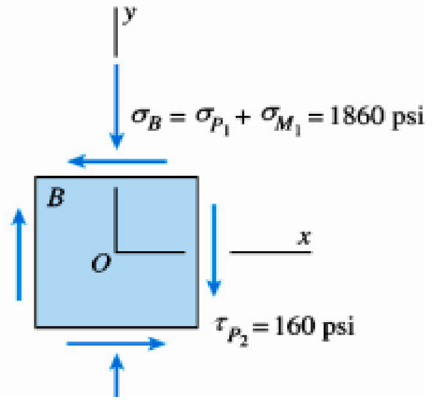
$\sigma_1 = \sigma_x$ and $\sigma_2 = \sigma_y = -4090psi$

And the maximum in-plane shear stress is

$\tau_{max} = (\sigma_1 - \sigma_2) / 2 = ½ (4090psi) = 2050psi$

The maximum out-of-plane shear stress has the same magnitude.




**Point B:**

Here the compressive stress in the ***y*** direction is

$\sigma_B = \sigma_{P1} + \sigma_{M1}$

$\sigma_B$ = 295psi + 1564psi = 1860psi (compression)

And the shear stress is

$\tau_B = \tau_{P2} = 160psi$

The shear stress acts leftward on the top face and downward on the ***x*** face of the element.

***Principal Stresses and Maximum Shear Stress***

**$\sigma_x$ =0**

**$\sigma_y$ = - $\sigma_B$ = - 1860psi**

**$\tau_{xy}$ = - $\tau_{P2}$ = - 160psi**

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Substituting $\sigma_{1,2}$ = - 930psi +/- 944psi

$\sigma_1$ = 14psi and $\sigma_2$ = - 1870psi

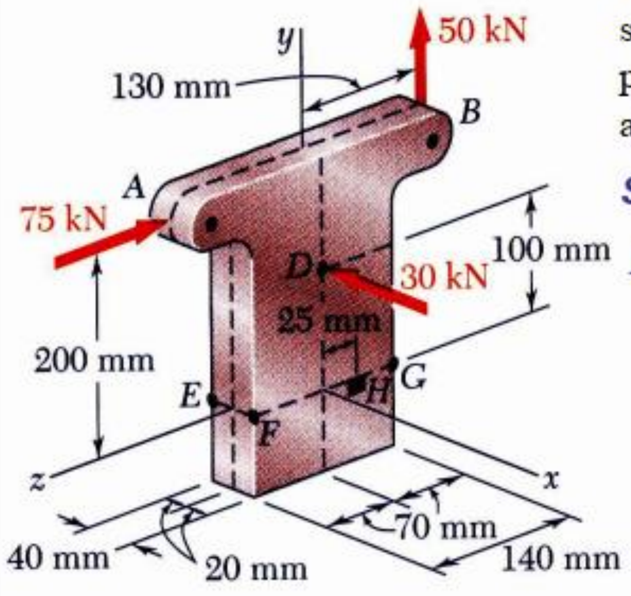
The maximum in-plane shear stresses can be obtained from the equation

Because the principal stresses have opposite signs, the maximum in-plane shear stresses are larger than the maximum out-of-plane shear stresses.

Then, $\tau_{max}$ = 944psi.

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = 944 \; psi$$

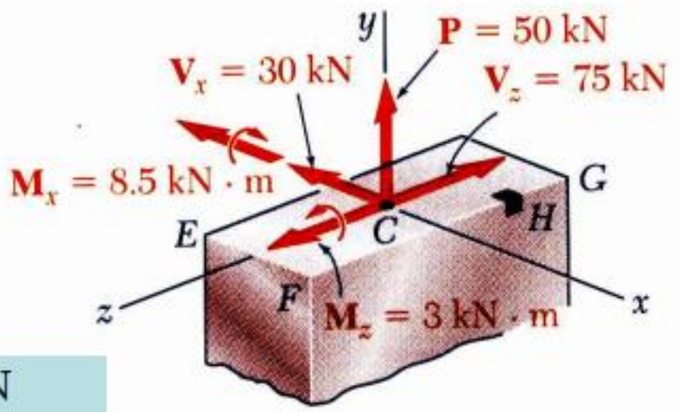
**Example**




Three forces are applied to a short steel post as shown. Determine the principle stresses, principal planes and maximum shearing stress at point *H*.

## Solution

***Determine internal forces in Section EFG.***




$$V_x = -30\,\text{kN} \quad P = 50\,\text{kN} \quad V_z = -75\,\text{kN}$$

$$M_x = (50\,\text{kN})(0.130\,\text{m}) - (75\,\text{kN})(0.200\,\text{m})$$

$$M_x = -8.5\,\text{kN}\cdot\text{m}$$

$$M_y = 0 \qquad M_z = (30\,\text{kN})(0.100\,\text{m}) = 3\,\text{kN}\cdot\text{m}$$

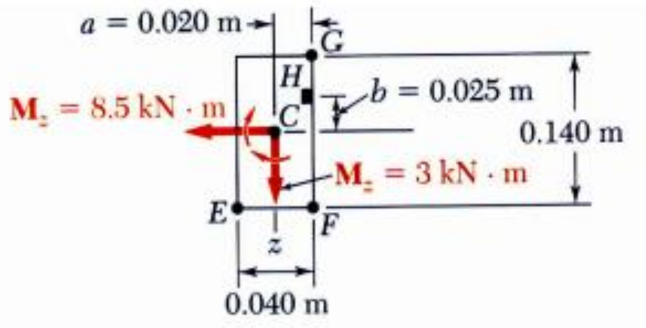
Note: Section properties,

$$A = (0.040\,\text{m})(0.140\,\text{m}) = 5.6\times10^{-3}\,\text{m}^2$$

$$I_x = \tfrac{1}{12}(0.040\,\text{m})(0.140\,\text{m})^3 = 9.15\times10^{-6}\,\text{m}^4$$

$$I_z = \tfrac{1}{12}(0.140\,\text{m})(0.040\,\text{m})^3 = 0.747\times10^{-6}\,\text{m}^4$$

***Evaluate the stresses at H.***




***Normal stress at H.***

$$\sigma_y = +\frac{P}{A} + \frac{|M_z|a}{I_z} - \frac{|M_x|b}{I_x}$$

$$= \frac{50\,\text{kN}}{5.6\times10^{-3}\,\text{m}^2} + \frac{(3\,\text{kN}\cdot\text{m})(0.020\,\text{m})}{0.747\times10^{-6}\,\text{m}^4} - \frac{(8.5\,\text{kN}\cdot\text{m})(0.025\,\text{m})}{9.15\times10^{-6}\,\text{m}^4}$$

$$= (8.93 + 80.3 - 23.2)\,\text{MPa} = 66.0\,\text{MPa}$$

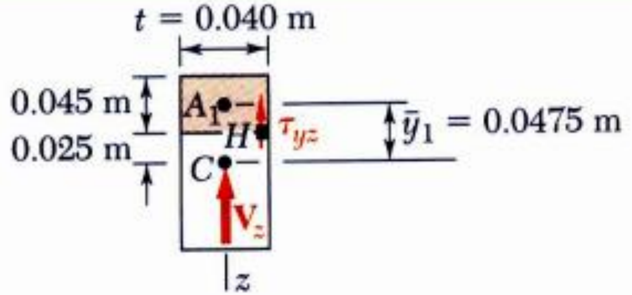
***Shear stress at H.***

$$Q = A_1\bar{y}_1 = [(0.040\,\text{m})(0.045\,\text{m})](0.0475\,\text{m})$$

$$= 85.5\times10^{-6}\,\text{m}^3$$

$$\tau_{yz} = \frac{V_z Q}{I_x t} = \frac{(75\,\text{kN})(85.5\times10^{-6}\,\text{m}^3)}{(9.15\times10^{-6}\,\text{m}^4)(0.040\,\text{m})}$$

$$= 17.52\,\text{MPa}$$



***Calculate principal stresses and maximum shearing stress.***

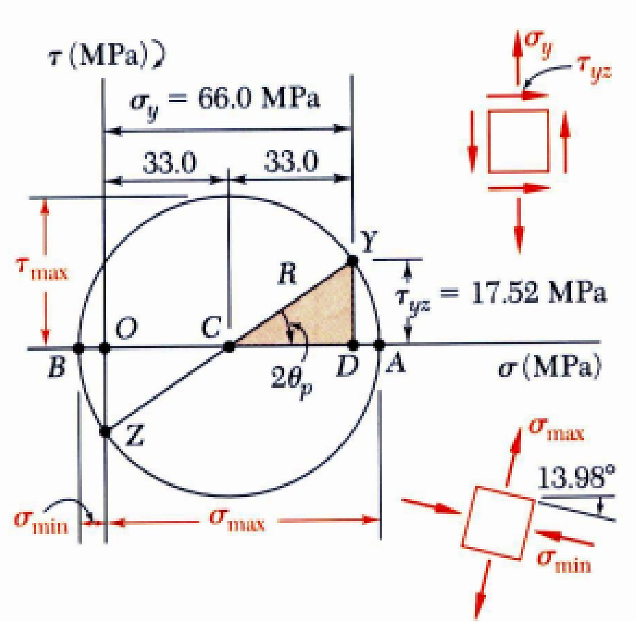
$$\tau_{max} = R = \sqrt{33.0^2 + 17.52^2} = 37.4\,\text{MPa}$$

$$\sigma_{max} = OC + R = 33.0 + 37.4 = 70.4\,\text{MPa}$$

$$\sigma_{min} = OC - R = 33.0 - 37.4 = -7.4\,\text{MPa}$$

$$\tan 2\theta_p = \frac{CY}{CD} = \frac{17.52}{33.0} \quad 2\theta_p = 27.96^\circ$$

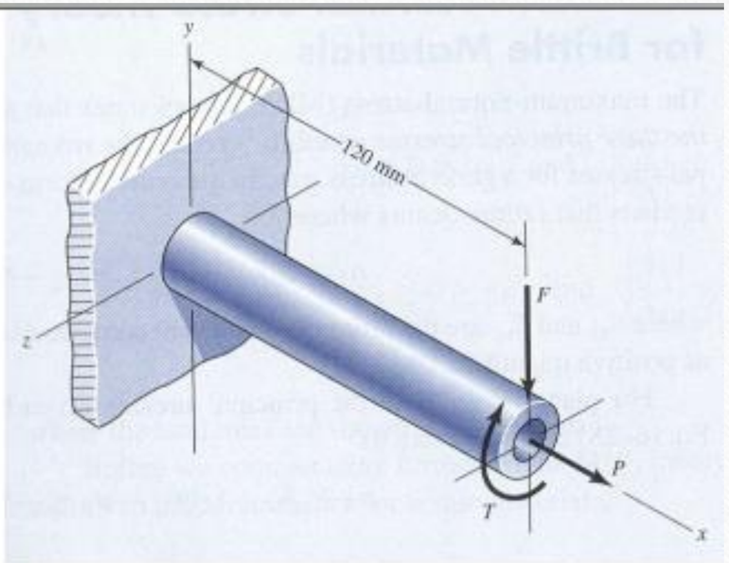
$$\theta_p = 13.98^\circ$$




$$\tau_{max} = 37.4\,\text{MPa}$$
$$\sigma_{max} = 70.4\,\text{MPa}$$
$$\sigma_{min} = -7.4\,\text{MPa}$$
$$\theta_p = 13.98^\circ$$

Example

The cantilever tube shown is to be made of 2014 aluminum alloy treated to obtain a specified minimum yield strength of 276MPa. We wish to select a stock size tube (according to the table below). Using a design factor of n=4.




The bending load is ***F=1.75kN***, the axial tension is ***P=9.0kN*** and the torsion is ***T=72N.m***. What is the realized factor of safety?

***Consider the critical area ( top surface).***

$$\sigma_{VM} \le \frac{S_y}{n} = \frac{0.276}{4} GPa = 0.0690GPa$$

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I}$$

Maximum bending moment = 120F

$$\sigma_x = \frac{9kN}{A} + \frac{120mm \times 1.75kN x \left(\frac{d}{2}\right)}{I}$$

$$\tau_{zx} = \frac{Tr}{J} = \frac{72 \times \left(\frac{d}{2}\right)}{J} = \frac{36d}{J}$$

$$\sigma_{VM} = \left(\sigma_x^2 + 3\tau_{zx}^2\right)^{1/2}$$

For the dimensions of that tube

$$n = \frac{S_y}{\sigma_{VM}} = \frac{0.276}{0.06043} = 4.57$$

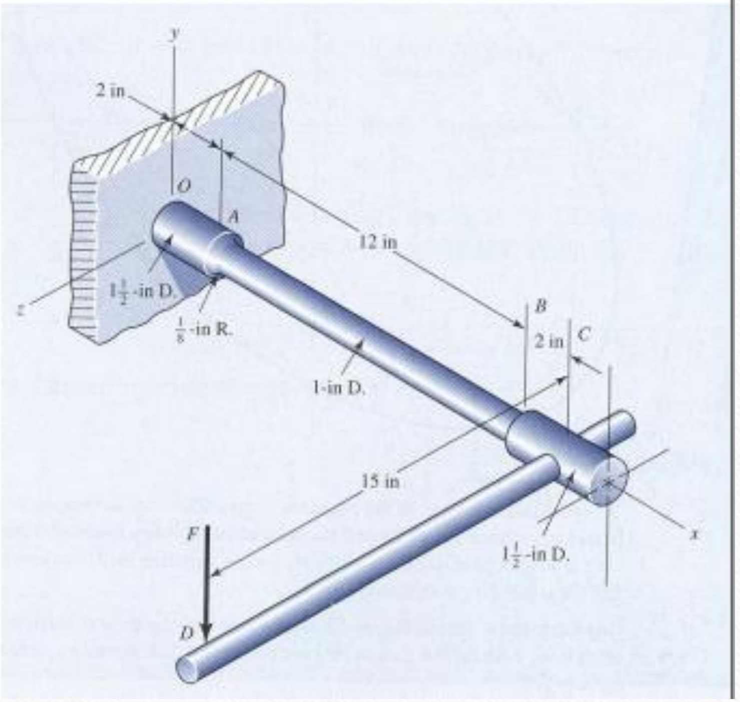
$w_a$ = unit weight of aluminum tubing, lbf/ft
$w_s$ = unit weight of steel tubing, lbf/ft
$m$ = unit mass, kg/m
$A$ = area, $in^2$ ($cm^2$)
$I$ = second moment of area, $in^4$ ($cm^4$)
$J$ = second polar moment of area, $in^4$ ($cm^4$)
$k$ = radius of gyration, in (cm)
$Z$ = section modulus, $in^3$ ($cm^3$)
$d, t$ = size (OD) and thickness, in (mm)

| Size, in | $w_a$ | $w_s$ | A | I | k | Z | J |
|---|---|---|---|---|---|---|---|
| $1 \times \frac{1}{8}$ | 0.416 | 1.128 | 0.344 | 0.034 | 0.313 | 0.067 | 0.067 |
| $1 \times \frac{1}{4}$ | 0.713 | 2.003 | 0.589 | 0.046 | 0.280 | 0.092 | 0.092 |
| $1\frac{1}{2} \times \frac{1}{8}$ | 0.653 | 1.769 | 0.540 | 0.129 | 0.488 | 0.172 | 0.257 |
| $1\frac{1}{2} \times \frac{1}{4}$ | 1.188 | 3.338 | 0.982 | 0.199 | 0.451 | 0.266 | 0.399 |
| $2 \times \frac{1}{8}$ | 0.891 | 2.670 | 0.736 | 0.325 | 0.664 | 0.325 | 0.650 |
| $2 \times \frac{1}{4}$ | 1.663 | 4.673 | 1.374 | 0.537 | 0.625 | 0.537 | 1.074 |
| $2\frac{1}{2} \times \frac{1}{8}$ | 1.129 | 3.050 | 0.933 | 0.660 | 0.841 | 0.528 | 1.319 |
| $2\frac{1}{2} \times \frac{1}{4}$ | 2.138 | 6.008 | 1.767 | 1.132 | 0.800 | 0.906 | 2.276 |
| $3 \times \frac{1}{4}$ | 2.614 | 7.343 | 2.160 | 2.059 | 0.976 | 1.373 | 4.117 |
| $3 \times \frac{3}{8}$ | 3.742 | 10.51 | 3.093 | 2.718 | 0.938 | 1.812 | 5.436 |
| $4 \times \frac{3}{16}$ | 2.717 | 7.654 | 2.246 | 4.090 | 1.350 | 2.045 | 8.180 |
| $4 \times \frac{3}{8}$ | 5.167 | 14.52 | 4.271 | 7.090 | 1.289 | 3.544 | 14.180 |

| Size, mm | m | A | I | k | Z | J |
|---|---|---|---|---|---|---|
| 12 × 2 | 0.490 | 0.628 | 0.082 | 0.361 | 0.136 | 0.163 |
| 16 × 2 | 0.687 | 0.879 | 0.220 | 0.500 | 0.275 | 0.440 |
| 16 × 3 | 0.956 | 1.225 | 0.273 | 0.472 | 0.341 | 0.545 |
| 20 × 4 | 1.569 | 2.010 | 0.684 | 0.583 | 0.684 | 1.367 |
| 25 × 4 | 2.060 | 2.638 | 1.508 | 0.756 | 1.206 | 3.015 |
| 25 × 5 | 2.452 | 3.140 | 1.669 | 0.729 | 1.336 | 3.338 |
| 30 × 4 | 2.550 | 3.266 | 2.827 | 0.930 | 1.885 | 5.652 |
| 30 × 5 | 3.065 | 3.925 | 3.192 | 0.901 | 2.128 | 6.381 |
| 42 × 4 | 3.727 | 4.773 | 8.717 | 1.351 | 4.151 | 17.430 |
| 42 × 5 | 4.536 | 5.809 | 10.130 | 1.320 | 4.825 | 20.255 |
| 50 × 4 | 4.512 | 5.778 | 15.409 | 1.632 | 6.164 | 30.810 |
| 50 × 5 | 5.517 | 7.065 | 18.118 | 1.601 | 7.247 | 36.226 |

Example

A certain force *F* is applied at *D* near the end of the *15-in* lever, which is similar to a socket wrench. The bar *OABC* is made of *AISI 1035* steel, forged and heat treated so that it has a minimum (ASTM) yield strength of *81kpsi*. Find the force (F) required to initiate yielding. Assume that the lever *DC* will not yield and that there is no stress concentration at *A*.




Solution:

1) Find the critical section

The critical sections will be either point *A* or Point *O*. As the moment of inertia varies with $r^4$ then point *A* in the *1in* diameter is the weakest section.

2) Determine the stresses at the critical section

$$\sigma_x = \frac{My}{I} = \frac{M\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = \frac{32 \times F \times 14in}{\pi d^3} = 142.6F$$

3) Chose the failure criteria.

The AISI 1035 is a ductile material. Hence, we need to employ the distortion-energy theory.

$$\tau_{zx} = \frac{Tr}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16 \times F \times 15in}{\pi (1in)^3} = 76.4F$$

$$\sigma_{VM} = \sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2} - \sigma_x\sigma_y + 3\tau_{xy}^2} = \sqrt{\sigma_x^{\ 2} + 3\tau_{zx}^2} = 194.5F$$

$$F = \frac{S_y}{\sigma_{VM}} = \frac{81000}{194.5} = 416lbf$$

Apply the MSS theory. For a point undergoing plane stress with only one non-zero normal stress and one shear stress, the two non-zero principal stresses ($\sigma_A$ and $\sigma_B$) will have opposite signs (Case 2).

$$\tau_{\max} = \frac{\sigma_A - \sigma_B}{2} = \frac{S_y}{2} = \pm\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{zx}^2}$$

$$\sigma_A - \sigma_B \geq S_y = 2\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{\sigma_x^{\ 2} + 4\tau_{zx}^2}$$

$$81000 = \left((142.6F)^2 + 4 \times (76.4F)^2\right)^{1/2}$$

$$F = 388 lbf$$

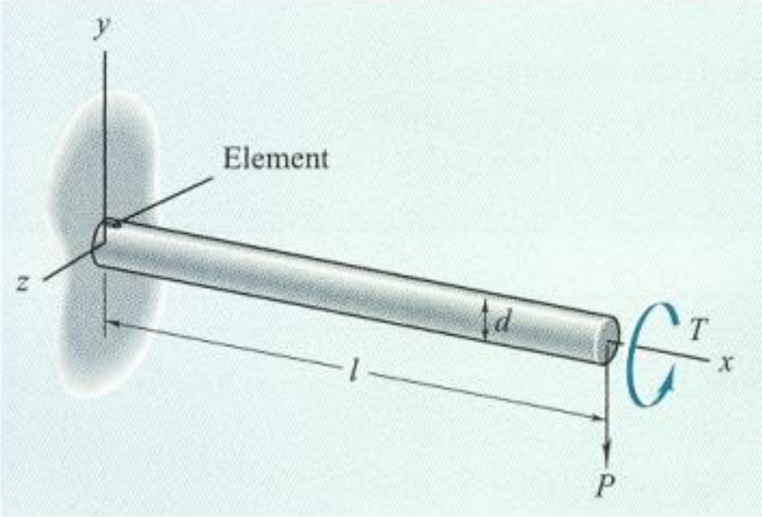
## Example

A round cantilever bar is subjected to torsion plus a transverse load at the free end. The bar is made of a ductile material having a yield strength of ***50000psi***. The transverse force (***P***) is ***500lb*** and the torque is ***1000lb-in*** applied to the free end. The bar is ***5in long (L)*** and a ***safety factor of 2*** is assumed. Transverse shear can be neglected. Determine the minimum diameter to avoid yielding using both MSS and DET criteria.

***Solution***

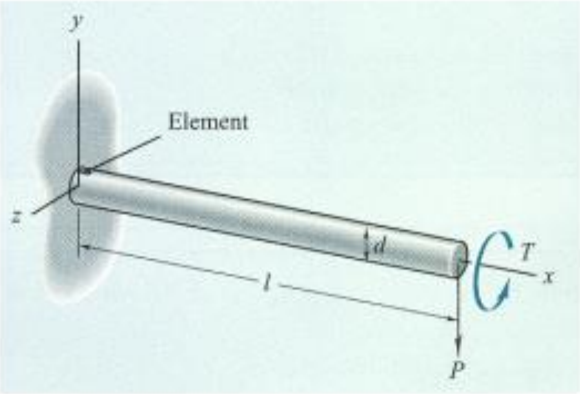
1) Determine the critical section

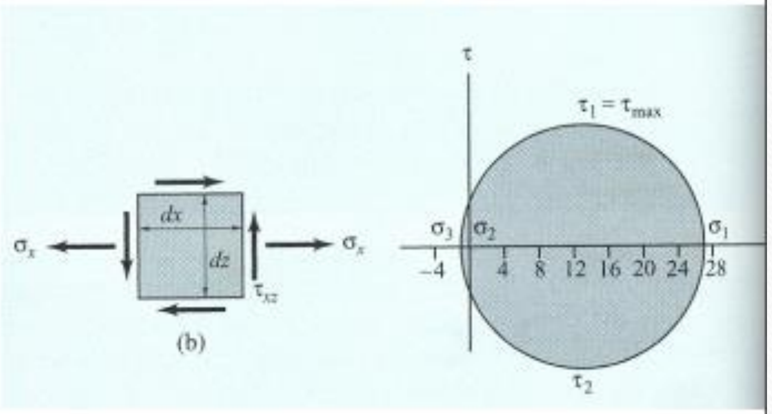
The critical section occurs at the wall.



$$\sigma_x = \frac{Mc}{I} = \frac{PL\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = \frac{32PL}{\pi d^3}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3}$$




$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \left(\frac{\sigma_x}{2}\right) \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{16PL}{\pi d^3} \pm \sqrt{\left(\frac{16PL}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[PL \pm \sqrt{(PL)^2 + T^2}\right]$$

$$\sigma_{1,2} = \frac{16}{\pi d^3}\left[500 \times 5 \pm \sqrt{(500 \times 5)^2 + 1000^2}\right]$$

$$\sigma_1 = \frac{26450}{d^3} \qquad \sigma_2 = -\frac{980.8}{d^3}$$

The stresses are in the wrong order.. Rearranged to

$$\sigma_1 = \frac{26450}{d^3} \qquad \sigma_3 = -\frac{980.8}{d^3}$$

MSS

$$\tau_{MAX} = \frac{\sigma_1 - \sigma_3}{2} = \frac{26450 - (-980.8)}{2d^3} = \frac{13715.4}{d^3}$$

$$\sigma_1 - \sigma_3 = 2\tau_{MAX} \leq \frac{S_y}{n} = \frac{50000}{2} = 25{,}000$$

$$d \geq 1.031\ in$$

DET

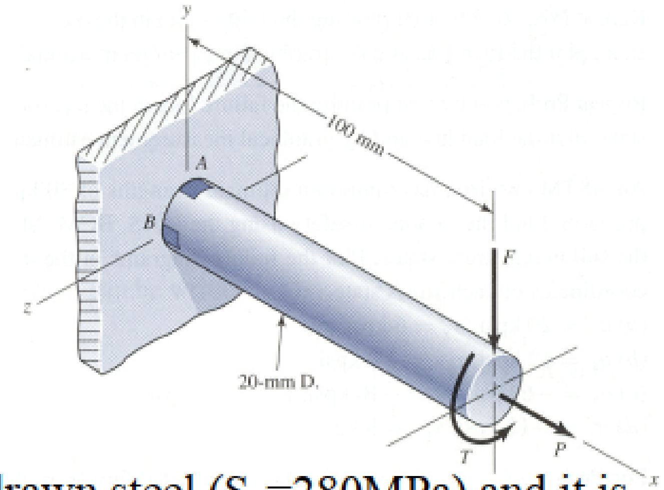
$$\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = \sqrt{\left(\frac{26450}{d^3}\right)^2 + \left(-\frac{980.8}{d^3}\right)^2 - \left(\frac{26450}{d^3}\right)\left(-\frac{980.8}{d^3}\right)}$$

$$\sigma_{VM} = \frac{26950}{d^3} \leq \frac{S_y}{n} = \frac{50000}{2}$$

$$d \geq 1.025 in$$

Example

The factor of safety for a machine element depends on the particular point selected for the analysis. Based upon the DET theory, determine the safety factor for points A and B.




This bar is made of AISI 1006 cold-drawn steel ($S_y$=280MPa) and it is loaded by the forces F=0.55kN, P=8.0kN and T=30N.m

***Solution:***

Point A

$$\sigma_x = \frac{Mc}{I} + \frac{P}{Area} = \frac{Fl\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} + \frac{P}{\frac{\pi d^2}{4}} = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2}$$

$$\sigma_x = \frac{32(0.55)(10^3)(0.1)}{\pi(0.02)^3} + \frac{4(8)(10^3)}{\pi(0.02)^2} = 95.49MPa$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020)^3} = 19.10MPa$$

$$\sigma_{VM} = \sqrt{\left(\sigma_x^2 + 3\tau_{xy}^2\right)} = \left[95.49^2 + 3(19.1)^2\right]^{1/2} = 101.1MPa$$

$$n = \frac{S_y}{\sigma_{VM}} = \frac{280}{101.1} = 2.77$$

Point B

$$\sigma_x = \frac{4P}{\pi d^2} = \frac{4(8)(10^3)}{\pi(0.02)^2} = 25.47MPa$$

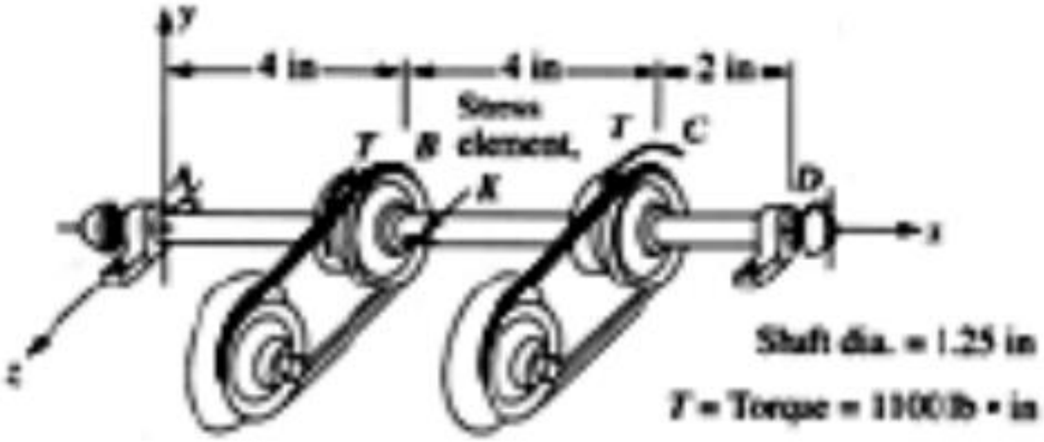
$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4V}{3A} = \frac{16(30)}{\pi(0.02)^3} + \frac{4(0.55)(10^3)}{3\left(\frac{\pi}{4}\right)(0.02)^2} = 21.43MPa$$

$$\sigma_{VM} = \left[25.47^2 + 3(21.43)^2\right]^{1/2} = 45.02MPa$$

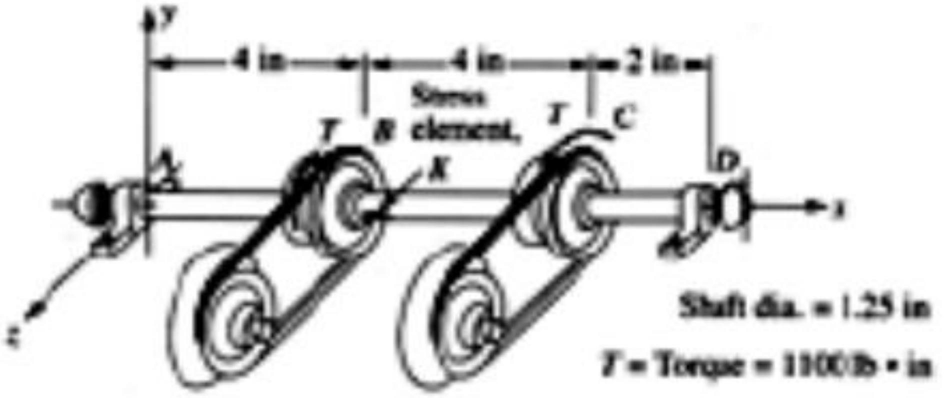
$$n = \frac{280}{45.02} = 6.22$$

## Example

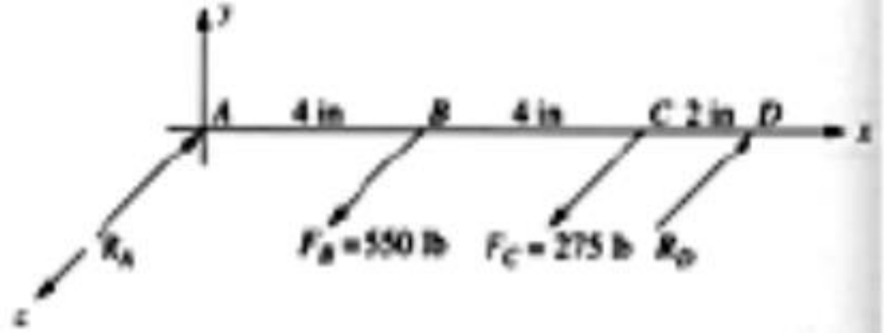
The shaft shown in the figure below is supported by two bearings and carries two V-belt sheaves. The tensions in the belts exert horizontal forces on the shaft, tending to bend it in the x-z plane. Sheaves B exerts a clockwise torque on the shaft when viewed towards the origin of the coordinate system along the x-axis. Sheaves C exerts an equal but opposite torque on the shaft. For the loading conditions shown, determine the principal stresses and the safety factor on the element K, located on the surface of the shaft (on the positive z-side), just to the right of sheave B. Consider that the shaft is made of a steel of a yield strength of 81ksi



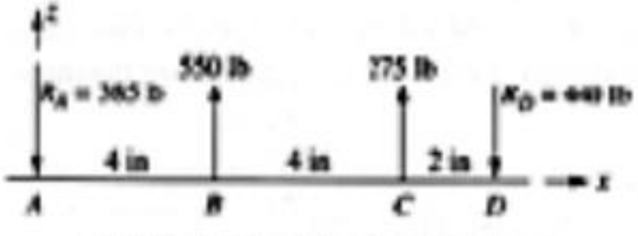

(a) Pictorial view of shaft



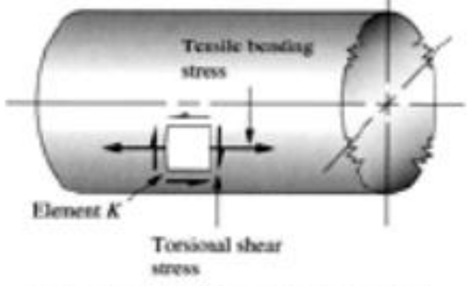

(a) Pictorial view of shaft



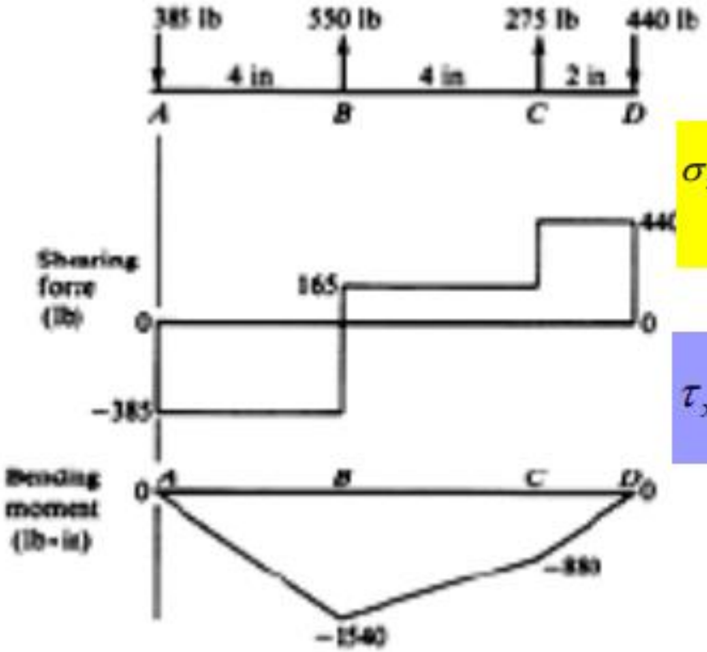

(b) Forces acting on shaft at B and C caused by belt drives




(c) Normal view of forces on shaft in x-z plane with reactions at bearings




(d) Enlarged view of element K on front of shaft

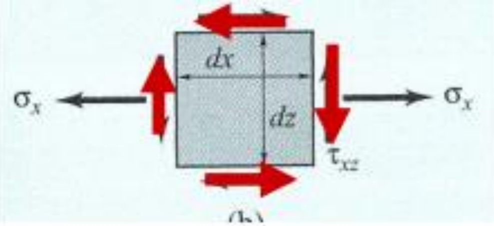



Shearing force = 165lb

Bending Moment = -1540lb-in

$$\sigma_x = -\frac{Mc}{I} = -\frac{M(r)}{\frac{\pi r^4}{4}} = -4\frac{-1540}{\pi(0.625)^3} = 8.031ksi$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{2T}{\pi r^3} = \frac{2(1100)}{\pi(0.625)^3} = 2.868ksi$$




$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \left(\frac{\sigma_x}{2}\right) \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{8.03}{2} \pm \sqrt{\left(\frac{8.03}{2}\right)^2 + (2.868)^2}$$

$$\sigma_1 = 8.95ksi \qquad \sigma_2 = -0.92ksi$$

$$MSS.....\tau_{Max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{8.95 - (-0.92)}{2} = 4.935ksi$$

$$Safety..Factor = n = \frac{S_Y/2}{\tau_{Max}} = \frac{81/2}{4.935} = 8.2$$

$$DET......\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = \sqrt{(8.95)^2 + (-0.92)^2 - (8.95)(-0.92)} = 9.44ksi$$

$$Safety..Factor = n = \frac{S_y}{\sigma_{VM}} = \frac{81}{9.44} = 8.58$$

## COMBINED BENDING AND DIRECT STRESSES

Consider the case of a column* subjected by a compressive load $P$ acting along the axis of the column as shown in Fig. 9.1. This load will cause a direct compressive stress whose intensity will be uniform across the cross-section of the column.

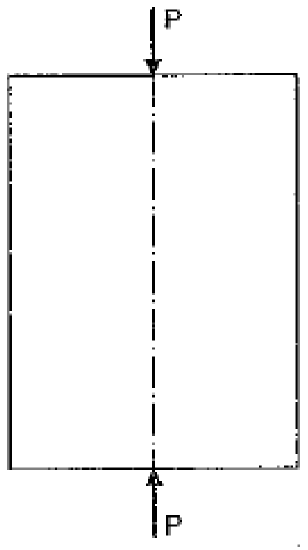


Fig. 9.1

Let $\sigma_0$ = Intensity of the stress

$A$ = Area of cross-section

$P$ = Load acting on the column.

Then stress,

$$\sigma_0 = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

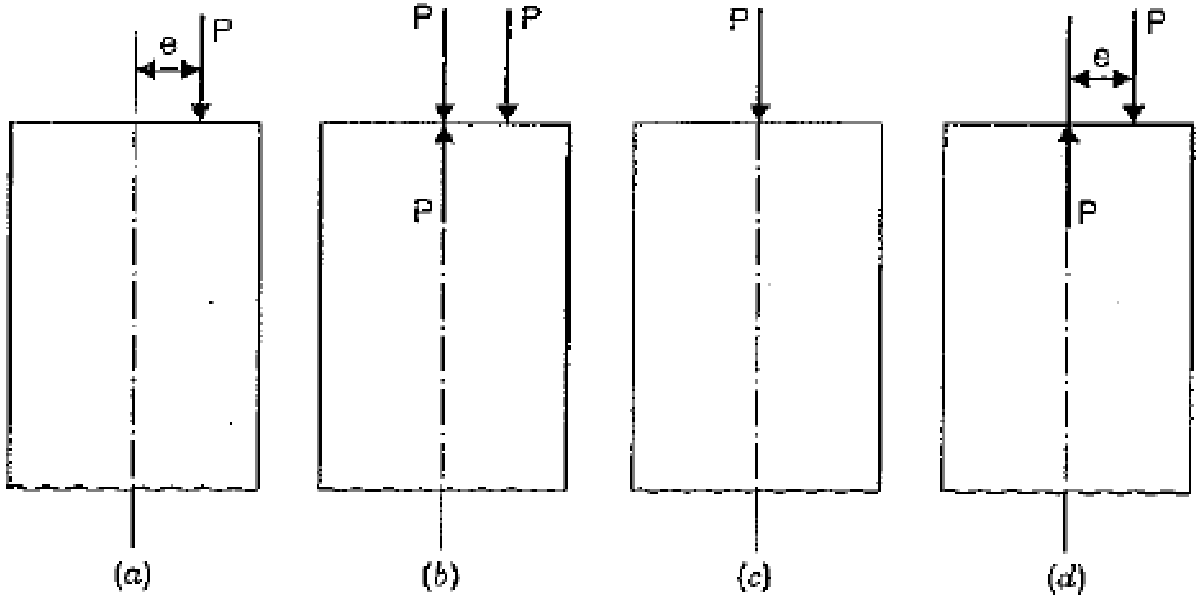
Now consider the case of a column subjected by a compressive load $P$ whose line of action is at a distance of '$e$' from the axis of the column as shown in Fig. 9.2 ($a$). Here '$e$' is known as eccentricity of the load. The eccentric load shown in Fig. 9.2 ($a$) will cause direct stress and bending stress. This is proved as discussed below :

1. In Fig. 9.2 ($b$), we have applied, along the axis of the column, two equal and opposite forces $P$. Thus three forces are acting now on the column. One of the forces is shown in Fig. 9.2 ($c$) and the other two forces are shown in Fig. 9.2 ($d$).

2. The force shown in Fig. 9.2 ($c$) is acting along the axis of the column and hence this force will produce a direct stress.

3. The forces shown in Fig. 9.2 ($d$) will form a couple, whose moment will be $P \times e$. This couple will produce a bending stress.

Hence an eccentric* load will produce a direct stress as well as a bending stress. By adding these two stresses algebraically, a single resultant stress can be obtained.



## RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO AN ECCENTRIC LOAD

A column of rectangular section subjected to an eccentric load is shown in Fig. 9.3. Let the load is eccentric with respect to the axis *Y-Y* as shown in Fig. 9.3 (*b*). It is mentioned in Art. 9.2 that an eccentric load causes direct stress as well as bending stress. Let us calculate these stresses.

Let $P$ = Eccentric load on column

$e$ = Eccentricity of the load

$\sigma_0$ = Direct stress

$\sigma_b$ = Bending stress

$b$ = Width of column

$d$ = Depth of column

$\therefore$ Area of column section, $A = b \times d$

Now moment due to eccentric load $P$ is given by,

$$M = \text{Load} \times \text{eccentricity}$$

$$= P \times e$$

The direct stress ($\sigma_0$) is given by,

$$\sigma_0 = \frac{\text{Load } (P)}{\text{Area}} = \frac{P}{A} \qquad \ldots(i)$$

This stress is uniform along the cross-section of the column.

The bending stress $\sigma_b$ due to moment at any point of the column section at a distance $y$ from the neutral axis $Y$-$Y$ is given by

$$\frac{M}{I} = \frac{\sigma_b}{\pm y}$$

$$\therefore \quad \sigma_b = \pm \frac{M}{I} \times y \quad \text{...(ii)}$$

where $I$ = Moment of inertia of the column section about the neutral axis $Y\text{-}Y = \frac{d \cdot b^3}{12}$

Substituting the value of $I$ in equation (*ii*), we get

$$\sigma_b = \pm \frac{M}{\frac{d \cdot b^3}{12}} \times y = \pm \frac{12\,M}{d \cdot b^3} \times y$$

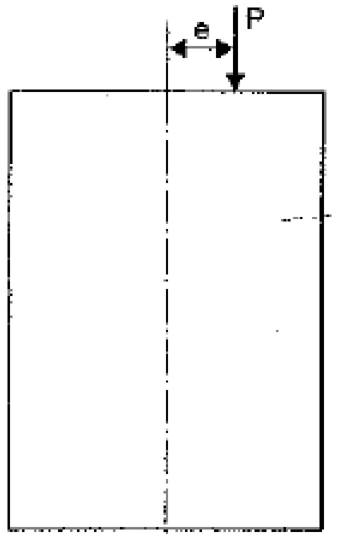
The bending stress depends upon the value of $y$ from the axis $Y\text{-}Y$.

The bending stress at the extreme is obtained by substituting $y = \frac{b}{2}$ in the above equation.

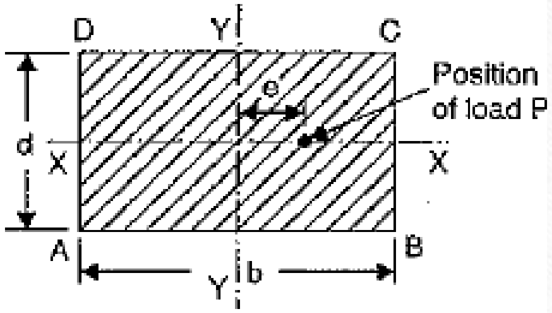
$$\therefore \quad \sigma_b = \pm \frac{12\,M}{d \cdot b^3} \times \frac{b}{2} = \pm \frac{6\,M}{d \cdot b^2}$$

$$= \pm \frac{6\,P \times e}{d \cdot b^2} \quad (\because \quad M = P \times e)$$

$$= \pm \frac{6\,P \times e}{d \cdot b \cdot b} = \pm \frac{6\,P \times e}{A \times b}$$



Elevation
(*a*)



Plan
(*b*)

($\because$ Area = $b \times d = A$)

The resultant stress at any point will be the algebraic sum of direct stress and bending stress.

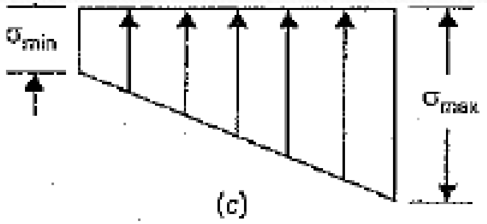



Fig. 9.3

If $y$ is taken positive on the same side of $Y$-$Y$ as the load, then bending stress will be of the same type as the direct stress. Here direct stress is compressive and hence bending stress will also be compressive towards the right of the axis $Y$-$Y$. Similarly bending stress will be tensile towards the left of the axis $Y$-$Y$. Taking compressive stress as positive and tensile stress as negative we can find the maximum and minimum stress at the extremities of the section. The stress will be maximum along layer $BC$ and minimum along layer $AD$.

Let $\sigma_{max}$ = Maximum stress (*i.e.*, stress along $BC$)

$\sigma_{min}$ = Minimum stress (*i.e.*, stress along $AD$)

Then $\sigma_{max}$ = Direct stress + Bending stress

$$= \sigma_0 + \sigma_b$$

$$= \frac{P}{A} + \frac{6P.e}{A.b} \quad \text{(Here bending stress is +ve)}$$

$$= \frac{P}{A}\left(1 + \frac{6 \times e}{b}\right) \quad \ldots(9.1)$$

and $\sigma_{min}$ = Direct stress – Bending stress

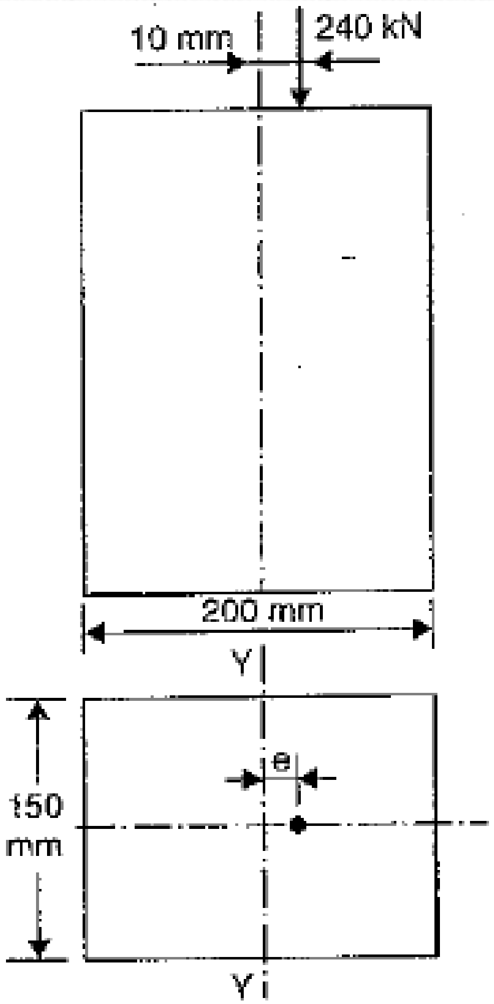
$$= \sigma_0 - \sigma_b$$

$$= \frac{P}{A} - \frac{6P.e}{A.b} = \frac{P}{A}\left(1 - \frac{6 \times e}{b}\right) \quad \ldots(9.2)$$

These stresses are shown in Fig. 9.3 (*c*). The resultant stress along the width of the column will vary by a straight line law.

If in equation (9.2), $\sigma_{min}$ is negative then the stress along the layer *AD* will be tensile. If $\sigma_{min}$ is zero then there will be no tensile stress along the width of the column. If $\sigma_{min}$ is positive then there will be only compressive stress along the width of the column.

**Problem 9.1.** *A rectangular column of width 200 mm and of thickness 150 mm carries a point load of 240 kN at an eccentricity of 10 mm as shown in Fig. 9.4 (i). Determine the maximum and minimum stresses on the section.*




**Sol. Given :**

Width, $b = 200$ mm

Thickness, $d = 150$ mm

∴ Area, $A = b \times d$

$= 200 \times 150 = 30000 \text{ mm}^2$

Eccentric load,

$P = 240$ kN

$= 240000$ N

Eccentricity,

$e = 10$ mm

Let $\sigma_{max}$ = Maximum stress, and

$\sigma_{min}$ = Minimum stress.

(*i*) Using equation (9.1), we get

Eccentricity,

$$e = 10 \text{ mm}$$

Let $\sigma_{max}$ = Maximum stress, and

$\sigma_{min}$ = Minimum stress.

(*i*) Using equation (9.1), we get

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6 \times e}{b}\right)$$

$$= \frac{240000}{30000}\left(1 + \frac{6 \times 10}{200}\right)$$

$$= 8(1 + 0.3) = \mathbf{10.4\ N/m^2}. \quad \textbf{Ans.}$$

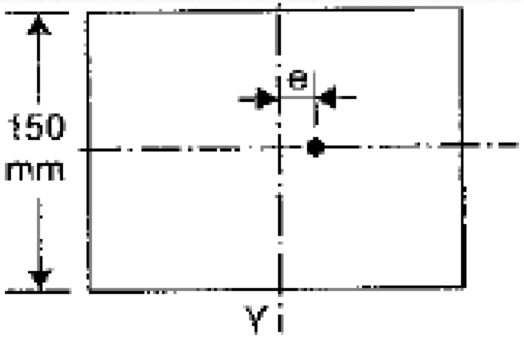



Fig. 9.4 (*i*)

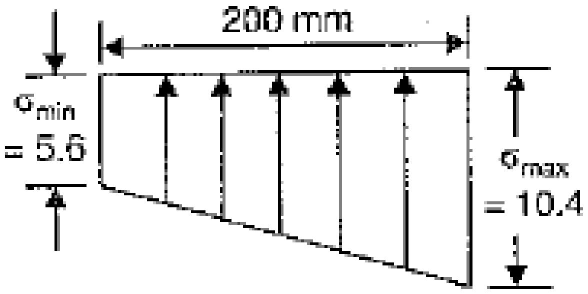



Fig. 9.4 (*ii*)

(*ii*) Using equation (9.2), we get

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6 \times e}{b}\right)$$

$$= \frac{240000}{30000}\left(1 - \frac{6 \times 10}{200}\right) = 8(1 - 0.3) = \mathbf{5.6\ N/mm^2}. \quad \textbf{Ans.}$$

These stresses are shown in Fig. 9.4 (*ii*).

*If in Problem 9.1, the minimum stress on the section is given zero then find the eccentricity of the point load of 240 kN acting on the rectangular column. Also calculate the corresponding maximum stress on the section.*

**Sol.** Given :

The data from Problem 9.1 is :

$b = 200$ mm, $d = 150$ mm, $P = 240000$ N, $A = 30000$ mm$^2$

Minimum stress,

$$\sigma_{min} = 0$$

Let $e$ = Eccentricity

Using equation (9.2), we get

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6 \times e}{b}\right)$$

or
$$0 = \frac{240000}{30000}\left(1 - \frac{6 \times e}{200}\right)$$

or
$$1 - \frac{6 \times e}{200} = 0 \quad \text{or} \quad 1 = \frac{6 \times e}{200}$$

$$\therefore \quad e = \frac{200}{6} = \mathbf{33.33\ mm.} \quad \textbf{Ans.}$$

Corresponding maximum stress is obtained by using equation (9.1).

$$\therefore \quad \sigma_{max} = \frac{P}{A}\left(1 + \frac{6 \times e}{b}\right)$$

$$= \frac{240000}{30000}\left(1 + \frac{6 \times \frac{200}{6}}{200}\right) = 8(1 + 1) = \mathbf{16\ N/mm^2}$$

The stresses are shown in Fig. 9.5.

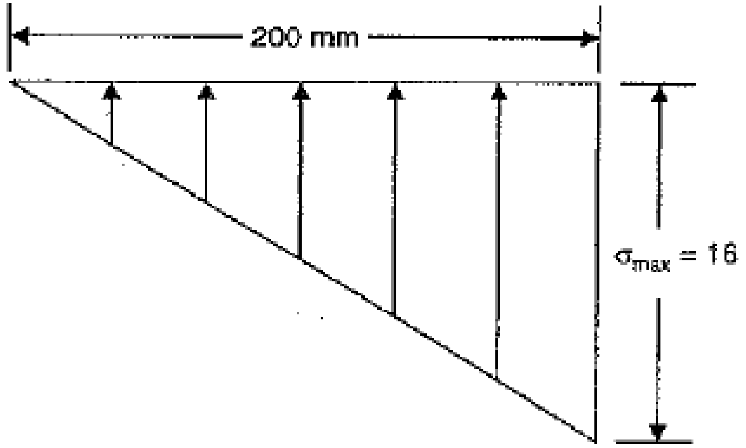



Fig. 9.5

.. *If in Problem 9.1, the eccentricity is given 50 mm instead of 10 mm then find the maximum and minimum stresses on the section. Also plot these stresses along the width of the section.*

**Sol.** Given :

The data from Problem 9.1 is :

$b = 200$ mm

$d = 150$ mm

$P = 240000$ N

$A = 30000$ mm$^2$

Eccentricity,

$e = 50$ mm

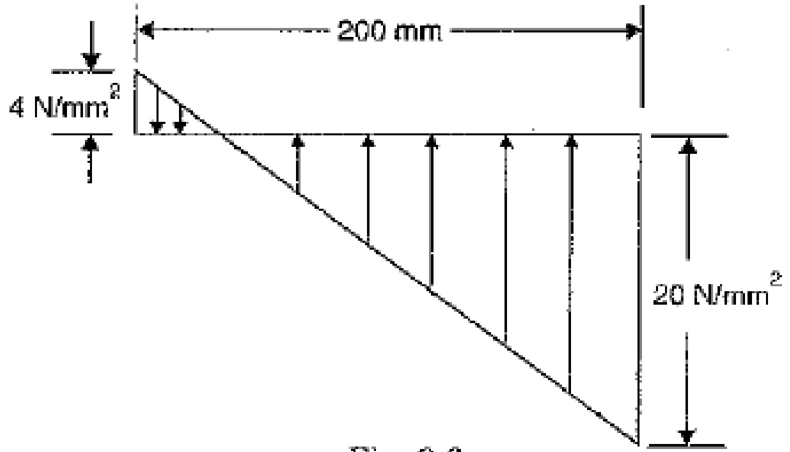



Fig. 9.6

(*i*) Maximum stress ($\sigma_{max}$) is given by equation (9.1) as

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6 \times e}{b}\right)$$

$$= \frac{240000}{30000}\left(1 + \frac{6 \times 50}{200}\right) = 8(1 + 1.5) = \mathbf{20\ N/mm^2}. \quad \textbf{Ans.}$$

(*ii*) Minimum stress ($\sigma_{min}$) is given by equation (9.2) as

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6 \times e}{b}\right)$$

$$= \frac{240000}{30000}\left(1 - \frac{6 \times 50}{200}\right) = 8(1 - 1.5) = -4 \text{ N/mm}^2. \quad \textbf{Ans.}$$

Negative sign means tensile stress.

The stresses are plotted as shown in Fig. 9.6.

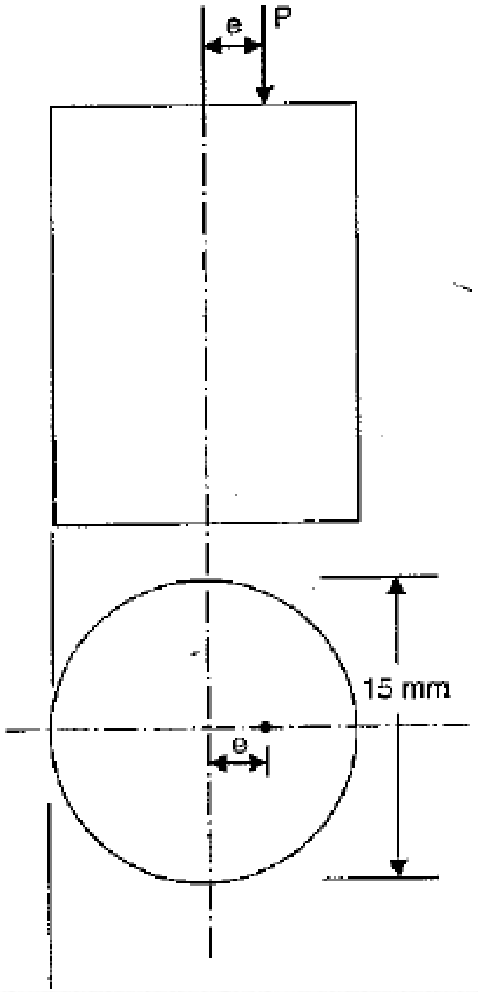
**Note.** From the above three problems, we have

(*i*) The minimum stress is zero when $e = \frac{200}{6}$ mm or $\frac{b}{6}$ mm (as $b = 200$). This is clear from Problem 9.2.

(*ii*) The minimum stress is +ve (*i.e.*, compressive) when $e < \frac{b}{6}$. This is clear from Problem 9.1 in which $e = 10$ mm which is less than $\frac{200}{6}$ (*i.e.*, 33.33).

(*iii*) The minimum stress is –ve (*i.e.*, tensile) when $e > \frac{b}{6}$. This is clear from Problem 9.3 in which $e = 50$ mm which is more than $\frac{200}{6}$ (*i.e.*, 33.33).

*The line of thrust, in a compression testing specimen 15 mm diameter, is parallel to the axis of the specimen but is displaced from it. Calculate the distance of the line of thrust from the axis when the maximum stress is 20% greater than the mean stress on a normal section.*




**Sol.** Given :

Diameter, $d = 15$ mm

$\therefore$ Area, $A = \frac{\pi}{4} \times 15^2$

$= 176.714$ mm$^2$

$\sigma_{max} = 20\%$ greater than mean

$= \frac{120}{100} \times$ mean stress

$= 1.2 \times$ mean stress.

Let $P$ = Compressive load on specimen

$e$ = Eccentricity

Mean stress $= \frac{\text{Load}}{\text{Area}} = \frac{P}{176.714}$ N/mm$^2$

We know that moment,

$$M = P \times e$$

Now bending stress is given by

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore \qquad \sigma_b = \frac{M}{I} \times y$$

$\therefore$ Maximum bending stress will be when $y = \pm \frac{d}{2}$.

Hence maximum bending stress is given by,

$$\sigma_b = \frac{M}{I} \times \left(\pm \frac{d}{2}\right)$$

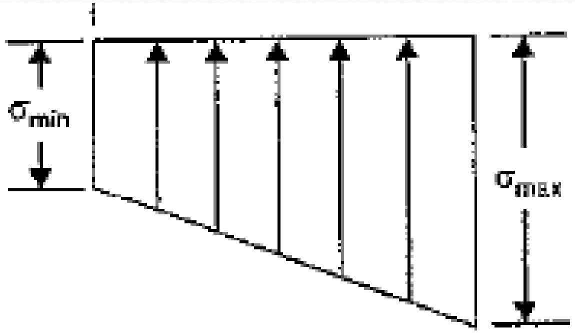



Fig. 9.7

$$= \pm \frac{M}{I} \times \frac{d}{2}$$

$$= \pm \frac{M}{\frac{\pi}{64} d^4} \times \frac{d}{2} \qquad \left(\because \ I = \frac{\pi}{64} d^4\right)$$

$$= \pm \frac{32\,M}{\pi d^3}$$

$$= \pm \frac{32\,P \times e}{\pi d^3} \qquad (\because \ M = P \times e)$$

Direct stress due to load is given by,

$$\sigma_0 = \frac{P}{A} = \frac{P}{176.714}$$

$\therefore$ Maximum stress = Direct stress × Bending stress

$$= \sigma_0 + \sigma_b$$

or

$$\sigma_{max} = \frac{P}{176.714} + \frac{32\,P \times e}{\pi d^3} \qquad \ldots(i)$$

But $\sigma_{max} = 1.2 \times$ Mean stress (given)

$$= 1.2 \times \frac{P}{176.714} \quad \ldots(ii) \quad \left(\because \text{ Mean stress} = \frac{P}{176.714}\right)$$

Equating equations (*i*) and (*ii*), we get

$$\frac{P}{176.714} + \frac{32\, P \times e}{\pi d^3} = 1.2 \times \frac{P}{176.714}$$

or

$$\frac{32 \times P \times e}{\pi d^3} = \frac{1.2\, P}{176.714} - \frac{P}{176.714} = \frac{0.2\, P}{176.714}$$

or

$$\frac{32 \times e}{\pi d^3} = \frac{0.2}{176.714} \quad \text{(Cancelling } P \text{ to both sides)}$$

$$\therefore \quad e = \frac{0.2 \times \pi \times d^3}{32 \times 176.714} = \frac{0.2 \times \pi \times 15^3}{32 \times 176.714} = \mathbf{0.375\ mm.} \quad \textbf{Ans.}$$

*A hollow rectangular column of external depth 1 m and external width 0.8 m is 10 cm thick. Calculate the maximum and minimum stress in the section of the column if a vertical load of 200 kN is acting with an eccentricity of 15 cm as shown in Fig. 9.8.*

**Sol. Given :**

External width, $B = 0.8 \text{ m} = 800 \text{ mm}$

External depth, $D = 1.0 \text{ m} = 1000 \text{ mm}$

Thickness of walls, $t = 10 \text{ cm} = 100 \text{ mm}$

Inner width, $b = B - 2 \times 100$

$= 800 - 200 = 600 \text{ mm}$

Inner depth, $d = D - 2 \times t$

$= 1000 - 2 \times 100 = 800 \text{ mm}$

∴ Area, $A = B \times D - b \times d$

$= 800 \times 1000 - 600 \times 800$

$= 800000 - 480000$

$= 320000 \text{ mm}^2$

M.O.I. about $Y$-$Y$ axis is given by,

$$I = \frac{1000 \times 800^3}{12} - \frac{800 \times 600^3}{12}$$

$$= 42.66 \times 10^9 - 14.4 \times 10^9$$

$$= 28.26 \times 10^9 \text{ mm}^4$$

Eccentric load, $P = 200 \text{ kN} = 200{,}000 \text{ N}$

Eccentricity, $e = 15 \text{ cm} = 150 \text{ mm}$

We know that the moment,

$$M = P \times e$$

$$= 200{,}000 \times 150$$

$$= 3000000 \text{ Nmm}$$

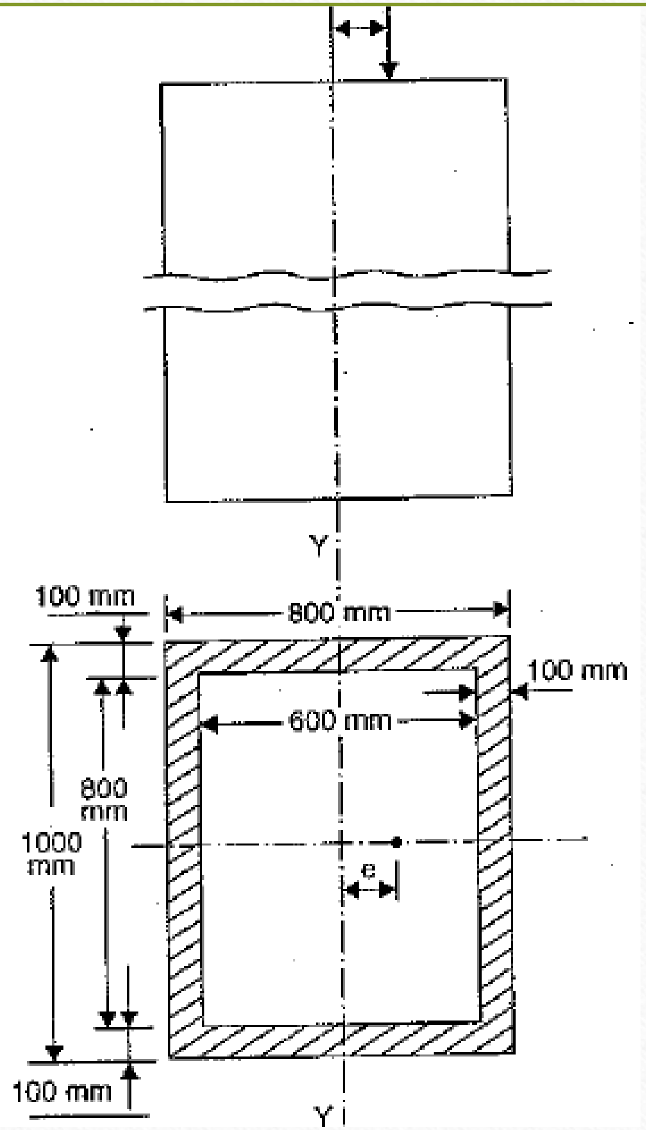
The bending stress is given by,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore \quad \sigma_b = \frac{M}{y} \times y$$

Maximum bending stress will be when

$$y = \pm 400$$



$$\therefore \quad \sigma_b = \frac{M}{I} \times (\pm 400)$$

$$= \pm \frac{30000000}{28.26 \times 10^9} \times 400$$

$$= \pm 0.4246 \text{ N/mm}^2$$

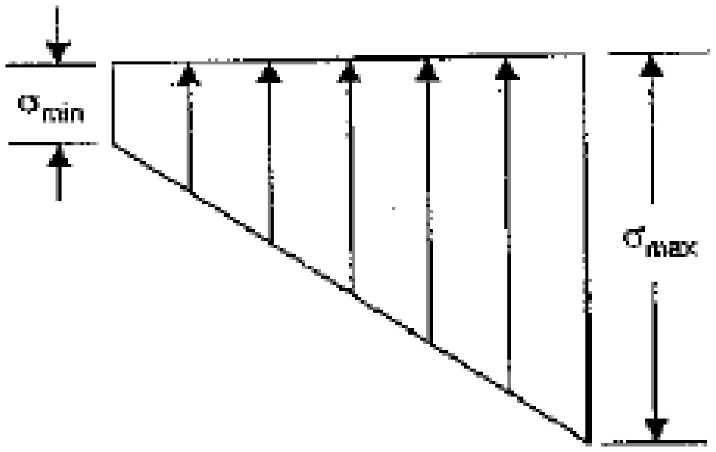



Fig. 9.8

Direct stress is given by,

$$\sigma_0 = \frac{P}{A} = \frac{200000}{320000}$$

$$= 0.625 \text{ N/mm}^2$$

$\therefore$ Maximum stress $= \sigma_0 + \sigma_b = 0.625 + 0.4246$

$= \mathbf{1.0496 \text{ N/mm}^2}$ **(Compressive). Ans.**

Minimum stress $= \sigma_0 - \sigma_b = 0.625 - 0.4246$

$= \mathbf{0.2004 \text{ N/mm}^2}$ **(Compressive). Ans.**

*A short column of external diameter 40 cm and internal diameter 20 cm carries an eccentric load of 80 kN. Find the greatest eccentricity which the load can have without producing tension on the cross-section.*

**Sol.** Given :

External dia., $D = 40 \text{ cm} = 400 \text{ mm}$

Internal dia., $d = 20 \text{ cm} = 200 \text{ mm}$

∴ Area of cross-section,

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(400^2 - 200^2) = 30000 \times \pi \text{ mm}^2$$

Moment of inertia

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(400^4 - 200^4) = 3.75 \times 10^8 \times \pi \text{ mm}^4$$

Eccentric load, $P = 80 \text{ kN} = 80000 \text{ N}$

Let $e$ = Eccentricity when there is no tension.

Now direct stress, $\sigma_0 = \frac{P}{A} = \frac{80000}{30000 \times \pi}$ ...(*i*)

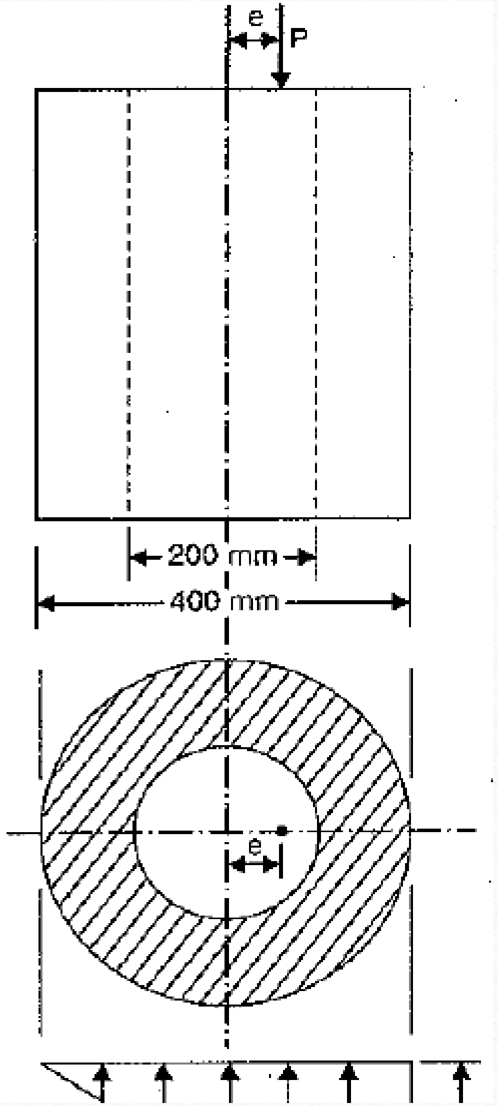
We know that moment,

$$M = P \times e = 80000 \times e$$

Now bending stress ($\sigma_b$) is given by

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore \quad \sigma_b = \frac{M \times y}{I}$$



The bending stress will be maximum when

$$y = \pm \frac{D}{2} = \pm \frac{400}{2} = \pm 200 \text{ mm}$$

$\therefore$ Maximum bending stress is given by,

$$\sigma_b = \frac{M \times (\pm 200)}{I} = \pm \frac{M \times 200}{I}$$

$$= \pm \frac{80000 \times e \times 200}{3.75 \times 10^8 \times \pi} \quad \ldots(ii)$$

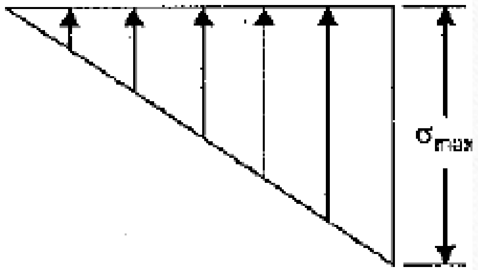


Fig. 9.9

Now minimum stress is given by,

$$\sigma_{min} = \sigma_0 - \sigma_b$$

$$= \frac{80000}{30000 \times \pi} - \frac{80000 \times e \times 200}{3.75 \times 10^8 \times \pi}$$

There will be no tension if $\sigma_{min} = 0$

$\therefore$ For no tension, we have

$$0 = \frac{80000}{30000 \times \pi} - \frac{80000 \times e \times 200}{3.75 \times 10^8 \times \pi}$$

There will be no tension if $\sigma_{min} = 0$

$\therefore$ For no tension, we have

$$0 = \frac{80000}{30000 \times \pi} - \frac{80000 \times e \times 200}{3.75 \times 10^8 \times \pi}$$

or

$$\frac{80000}{30000 \times \pi} = \frac{80000 \times e \times 200}{3.75 \times 10^8 \times \pi}$$

1. A hollow circular column having internal dia 400 mm and thickness 100 mm is used as a column. If slenderness ratio is 90. One end of column is restrained against position and direction while other end is restrained against position. Calculate actual length of the column.